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Title: How droplets nucleate and grow on liquids and liquid impregnated surfaces

While nucleation is most likely to occur at the liquid–air interface, the liquid viscosity plays a key role in determining the overall growth of droplets.
How droplets nucleate and grow on liquids and liquid impregnated surfaces†

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Condensation on liquids has been studied extensively in context of breath figure templating, materials synthesis and enhancing heat transfer using liquid impregnated surfaces. However, the mechanics of nucleation and growth on liquids remains unclear, especially on liquids that spread on the condensate. By examining the energy barriers of nucleation, we provide a framework to choose liquids that can lead to enhanced nucleation. We show that due to limits of vapor sorption within a liquid, nucleation is most favoured at the liquid–air interface and demonstrate that on spreading liquids, droplet submergence within the liquid occurs thereafter. We provide a direct visualization of the thin liquid profile that cloaks the condensed droplet on a liquid impregnated surface and elucidate the vapour transport mechanism in the liquid films. Finally, we show that although the viscosity of the liquid does not affect droplet nucleation, it plays a crucial role in droplet growth.

1. Introduction

Condensation of vapor on a surface can occur either in filmwise or dropwise mode. For many applications including power generation, water harvesting, desalination, and thermal management, dropwise condensation is preferred as it expedites the removal of the condensate from the surface resulting in a significant increase in condensation heat transfer.1–6 The magnitude of this increase is related to the three stages of a droplet “life” on a surface: nucleation, growth, and departure. Various routes of optimizing these aspects of dropwise condensation via surface engineering have been proposed. Functionalizing the surface to obtain a hydrophobic surface chemistry and combining it with nano/micro-texturing to achieve superhydrophobicity are some of the ways explored to reduce droplet adhesion as measured through contact angle hysteresis.5,7–9 In some cases, the reduced adhesion on superhydrophobic surfaces can also lead to micro-droplet ejection upon coalescence with neighbouring droplets.10–12 However, under high subcooling conditions, nucleation of a large density of nanoscale droplets within the texture of superhydrophobic surfaces leads to the formation of a liquid film on the surface.13–15 In order to overcome such challenges, an alternative surface design with composite solid–liquid materials has gained significant attention recently.16–22 Such composite materials consist of nano/micro textured low surface energy impregnating liquid with a liquid (henceforth also referred to as oil, and denoted with subscript ‘o’) immiscible with the fluid (henceforth denoted with subscript ‘w’) to be repelled. These liquid-impregnated surfaces (LIS) shed a wide variety of fluids22 with minimal contact angle hysteresis16–24 and have been recently shown to significantly increase dropwise condensation heat transfer of fluids with widely ranging surface tensions.21

The three stages of condensation (nucleation, growth, and departure) on LIS are greatly influenced by the properties of the impregnating liquid. It has been reported that the nucleation energy barrier for condensation is lowered19 and nucleation rates are enhanced25 on LIS, when compared to superhydrophobic surfaces with identical solid surface chemistry. However, the role of oil properties on nucleation remains unclear. Furthermore, an oil may “cloak” the condensing droplets if the spreading coefficient of the oil with respect to the droplet is positive, i.e. \( S_{ow(a)} = \gamma_{wa} - \gamma_{oa} - \gamma_{wo} > 0 \) (\( \gamma_{wa} \), \( \gamma_{oa} \), and \( \gamma_{wo} \) refer to the surface tension of the droplet, surface tension of the oil, and the interfacial tension between the oil-droplet, respectively), and this leads to a suppression of droplet growth.19 However, even in the presence of the cloaking mechanism, sustained growth of water droplets on a polymer,22 a pure solvent20 or solvent–polymer mixtures with solvent spreading coefficient \( S_{ow(a)} > 0 \) has been observed.27–30 These
contrasting observations highlight the need to understand the mechanism underlying droplet nucleation and growth on oils.

In this study, we use theoretical and experimental approaches to understand nucleation and growth of droplets on immiscible oils. Here, we have identified the different pathways for nucleation on LIS and clarified the nucleation energetic barriers associated with them using classical nucleation theory. Our results indicate that the nucleation energy barrier is significantly lowered within the oil as compared to nucleation in air for some oil–solid combinations – provided that the critical supersaturation is available. However, in a subcooled oil, the vapor–sorption process prevents the vapor to achieve supersaturation in the oil, so that the oil–air interface is the most favored site for nucleation. We investigate the mechanisms accompanying growth of droplets on cloaking oils, and used the cryogenic Focused Ion Beam-Scanning Electron Microscopy (cryo-FIB-SEM) to uncover phenomena such as the presence of submerged droplets within the oil, and the oil nanofilm profile around a condensed droplet on LIS. Finally, we have carried systematic investigation of the effect of oil viscosity on droplet coalescence and growth. Our results could provide important insights into the dynamics of condensation on liquids for applications such as breath figure templating,27-31 materials synthesis,32 and oil recovery by steam injection.33,34

2. Materials and methods

2.1 Preparation of silicon samples

Arrays of square microposts of 10 μm height (h), 10 μm width (a) and 10 μm edge-to-edge spacing (b) were patterned via photo-lithography and etched via deep reactive ion etching (DRIE) on two centimetre square silicon substrates (p-type (100), 650 μm thick). Thereafter the samples were cleaned using piranha solution and subsequently coated with OTS (octadecyltri-chlorosilane, Sigma Aldrich) using a solution deposition method. Silanization of the sample renders the surface hydrophobic and allows the oil to stably adhere to the surface in the presence of water.

2.2 Liquids used in the current study

The liquids used in the current study were silicone oils with a viscosity of 10 cSt (ρo = 935 kg m⁻³, Mₒ ~ 1250), 100 cSt (ρo = 960 kg m⁻³, Mₒ = 5970) and 1000 cSt (ρo = 970 kg m⁻³, Mₒ = 28 000) purchased from Sigma-Aldrich. Other liquids used in the study (tetradecane, hexadecane, 1-bromonaphthalene, 1-butyl-3-methylimidazolium bis(trifluoromethylsulfonyl)imide ([BMIm][Tf2N⁻]) were also purchased from Sigma Aldrich.

2.3 Preparation of impregnated samples

The silanized samples were dipped in a reservoir of the impregnating oil with a dip-coater (KSV Nima Multi Vessel Dip Coater). In order to prevent excess oil on the samples, they were withdrawn at a controlled velocity V such that the capillary number Ca = μoV/γoa was less than 10⁻⁴.35

2.4 Contact angle measurements

Contact angles of the oils on smooth OTS-coated silicon surfaces were measured in the presence of air, as well as water using a Ramé-Hart Model 500 Advanced Goniometer. The interfacial tension between the oils and water was measured using the pendant drop technique on the same device.

2.5 Apparatus for vapor absorption and condensation

An annular steel ring (diameter: 25.4 mm, height: 5 mm) was attached to a smooth silicon surface using an adhesive to hold the oil. The silicon surface was cleaned thoroughly with acetone and isopropanol before attaching to the ring and displayed partially wetting behaviour. After filling the holder with 0.5 ml of 10 cSt silicone oil, the setup was put on a peltier cooler. The peltier temperature was lowered below the room temperature (25 ± 1 °C) to 16 ± 1 °C, but kept above the dew point (13 ± 1 °C) to prevent supersaturation in the air near the setup. After two hours, the temperature of the peltier was lowered to 9 ± 1 °C. Condensation followed within seconds on the surface. Condensation on the oil was observed using a Zeiss AxioZoom microscope fitted with a ‘Plan APO-Z 1.5× lens’ and a polarizer at 260× magnification. The videos were recorded using a Nikon D-800 camera at a resolution of 1920 × 1080 and 30 fps. The schematic of the experimental setup is shown in the ESI Fig. S1(a).†

To determine if nucleation occurred within vapour-saturated oil that was subcooled, 10 cSt silicone oil was used as the test liquid. Deionized water in a flask was bubbled with the dry nitrogen gas to obtain vapour-saturated air. The vapor-saturated air was then bubbled through 15 ml of 10 cSt silicone oil kept in a beaker for three hours. The schematic of this setup is shown in ESI Fig. S1(b).† Thereafter, 10 ml of vapour-saturated silicone oil was extracted in an airtight glass vial of 10 ml capacity with a partially wetting surface. After insulating the glass vial sidewalls, it was cooled to a temperature of −2 °C for a period of three hours. It was made sure that no air bubble remained in the glass vial before it was subjected to cooling. The room temperature was measured as 20 °C and a room humidity of 60% implying a dew point of 12 °C. After three hours, 20 μl of solution was extracted from the glass vial and analysed using the dynamic light scattering (DLS) setup. DLS measurements were performed using DynaPro NanoStar™, capable of identifying droplets in the size range of 0.2–2500 nm hydrodynamic radius. DLS measurements were performed ten times for one extraction volume. The experiment was repeated using three separate samples extracted from the solution.

2.6 Experiment procedure for cryo-FIB-SEM

The Cryo-SEM technique was used to characterize the morphology of various samples. For cross-sectional imaging of oil substrates, we used a modification of the cryogenic Focused Ion Beam-Scanning Electron Microscopy (cryo-FIB-SEM) method, which has recently been used to image water drops,36 frost and ice growth37 and adhesion mechanisms on superhydrophobic and liquid impregnated surfaces.38.
experiments were performed and analyzed using this technique. In the first experiment, water was condensed on silicone oil. Customized copper stubs with holes (2 mm in width and depth) were fabricated to contain the oil. A copper stub was mounted to the cryo-shuttle transfer device and filled with the oil of known viscosity (10, 100 and 1000 cSt). To trigger condensation on silicone oil, the temperature of the entire assembly was decreased using a water ice bath cooled peltier element. The temperature of the stub was monitored using a thermocouple mounted in a hole mounted on the side on the cup. A decrease in oil volume within the stub hole was observed during condensation due to oil displacement by droplets, and spreading of oil on condensed droplets on the copper stub. Then, the stub was plunge-frozen in liquid nitrogen slush at −210°C. The sample vacuum transfer, thin metal film grounding, FIB-cutting, and SEM-imaging procedures are described elsewhere.36,37 The SEM images of the FIB-cross sections were taken at a 52-degree sample stage tilt.

In the second experiment, water droplets were condensed on LIS. Silicon substrates with micropost arrays as described in Section 2.1 were impregnated with silicone oils of 10, 100 and 1000 cSt of viscosity by the method as described in Section 2.3. Thereafter, condensation and the cryo-FIB-SEM analysis were performed with the same methodology as described before for the first experiment.

The identification of the individual phases of oil/water/platinum was obtained by virtue of the imaging contrast combined with in situ elemental analysis. In backscattered electron imaging, the contrast of individual phases correlates strongly with their density. Although the densities of water and silicone oils are of similar order, sufficient contrast was observed between these two liquid phases. To ensure proper interpretation of the two phases imaged in FIB milled cross-sections, elemental analysis was also performed using Energy-Dispersive X-ray Spectroscopy (EDS). As in our previous work, spectra corresponding to water consisted primarily of oxygen signals, while those corresponding to silicone oil also contained silicon and carbon peaks. To avoid electron beam heating damage to the cut surface, only point spectra outside of the area of interest were taken. Since the topic of elemental tagging of water and oil was covered in our previous work,36 the spectra were not saved for presentation.

2.7 Apparatus for condensation observation on LIS

Silicon samples with micropost arrays (described in Section 2.1) were impregnated with silicone oils (with a methodology described in Section 2.3). The samples were kept on a cooling block maintained at a constant temperature of 3 ± 1 °C. All the experiments were performed in an open environment under the same conditions (Room Temperature 20 ± 1 °C and dew point 12 ± 1 °C). The humidity near the sample was continuously monitored using a Sensirion KT-71 humidity sensor. Condensation on the surface was observed using a Zeiss AxiosZoom microscope fitted with a ‘Plan APO-Z 1.5× lens’ and a polarizer at 260× magnification. A Nikon D-800 camera was used to record videos at 1920 × 1080 and 30 fps. Videos were analyzed to evaluate droplet growth using ImageJ software.39 The schematic of the experimental setup is shown in the ESI section (Fig. S1(a)†).

2.8 Image analysis

From the videos, the frames were extracted for analysis of droplet mobility that was performed using ImageJ.39 Droplet areas were measured for all the droplets in a given frame and from these measurements, the area fraction \( A_{\text{fraction}} \) under condensation at a time \( t \) occupied by droplets was calculated by

\[
(A_{\text{fraction}})_t = \left( \frac{\sum_{i=1}^{N} (\text{Droplet Area})_i}{\text{Frame Area}} \right)_t
\]

The polydispersity or the size variation in droplet sizes in a frame at a time \( t \) was calculated as \( \text{polydispersity} = \frac{D_{w.t}}{D_{n.t}} \)

where \( D_{w.t} \) is the weight averaged diameter and given by

\[
D_{w.t} = \left( \frac{\sum_{i=1}^{N} (\text{Droplet Diameter})_i^2 \times (\text{Droplet Area})_i}{\sum_{i=1}^{N} (\text{Droplet Diameter})_i^2} \right)_t
\]

and \( D_{n.t} \) is the number-averaged diameter given by

\[
D_{n.t} = \left( \frac{\sum_{i=1}^{N} (\text{Droplet Diameter})_i \times (\text{Droplet Area})_i}{\sum_{i=1}^{N} (\text{Droplet Diameter})_i} \right)_t
\]

3. Results and discussion

3.1 Nucleation states and energetic barriers

On LIS, the impregnating oil can exist in one of the two states — with emergent post tops and fully submerged features.38 Depending on the state in which oil exists in LIS, nucleation can occur at the solid–air interface (Fig. 1a-State I), within the oil (homogeneous nucleation, Fig. 1a-State II), at the solid–oil interface (heterogeneous nucleation within oil, Fig. 1a-State III), or at the oil–air interface (heterogeneous nucleation on oil, Fig. 1a-State IV). In a recent work, it has been suggested that nucleation enhancement occurs on LIS compared to superhydrophobic surfaces because the presence of ‘high surface energy sites’ on the submerged solid surfaces results in significantly lower energy barrier in State III compared to State I.23 However, these observations could also be attributed to the nucleation at the oil–air interface (State IV), as it may also be energetically favourable compared to State I. Nucleation of water droplets on bulk oil surfaces is well known,36 and very large nucleation rates have also been observed on bulk oils.41,42 Although nucleation at solid–air4 and oil–air41–44 interfaces has been examined before, a comprehensive comparison of vapour nucleation in different possible states has been lacking. Such comparison can not only lead to understanding of the energy barriers for nucleation on oils compared to the solid surfaces, but could also be used to choose oils that can enhance or suppress nucleation rates on LIS. Here we examine the free energy barrier of nucleation for these different states in order to identify the most preferable nucleation pathway.

From Classical Nucleation Theory (CNT), the work of cluster formation (W) through nucleation is given as

\[
W(n) = -nkT \ln(SR) + E
\]

where \( n \) corresponds to the number of molecules in the cluster, SR is the supersaturation, \( k \) is the...
Boltzmann constant, and \(E\) corresponds to the total interfacial energy of the cluster. The number of molecules in the cluster \(n\) is related to the volume of the cluster as \(n = V/n_m\) where \(n_m\) corresponds to the volume of a single molecule of condensate, \(V = \pi R^3/3\) is the volume of the cluster with a radius of curvature \(R\) and \(\psi\) is a shape factor associated with the geometry of the cluster. The interfacial energy term \(E\) can be expressed as

\[
E = \psi \frac{3V}{n_m} \gamma = \psi \frac{3n_m}{\pi \psi} \frac{2/3}{\gamma} = \frac{n^{2/3}}{\beta} \gamma
\]

where, \(\gamma\) is the interfacial tension associated with the cluster and its environment. Thus the nucleation work function can be written as \(W(n) = -n_0 + n^{2/3} \beta\) where \(n_0 = kT \ln(SR)\) and \(\beta = \psi \pi [(3n_m)/(\pi \psi)]^{2/3}\gamma\). By minimizing the nucleation work with respect to \(n\), the work of formation \((W^* = 4\beta^2/27a^2)\) of a critical cluster size \(n^*\) (where \(n^* = [(2\beta)/(3a)]\)) is obtained. Alternatively, the energy barrier for nucleation in each state can be evaluated through the critical supersaturation \((SR^* = \exp[(2\beta)/(3kTn^{1/3})])\) required to cause nucleation of a critical cluster size \(n^*\). The critical supersaturation \(SR^*\) required to form a critical cluster of size \(n^*\) is only dependent upon the surface energy (\(\beta\) and hence \(E\)) at a given temperature. By comparing the surface energies (\(E\)) in each state (see ESI Note 1†), we can determine a regime map of states that is favorable for nucleation.

Comparing State I and State III, the nucleation within oil is preferable if

\[
\frac{E_{III}}{E_I} = \frac{\ln(SR_{III})}{\ln(SR_I)} = \left(\frac{\psi_2}{\psi_1}\right)^{1/3} \frac{\gamma_{wa}}{\gamma_{wo}} < 1
\]

Here \(\psi\) is the shape factor of form \(\psi = (2 + \cos \theta)(1 - \cos \theta)^2\). \(\psi_1\) and \(\psi_2\) are related to the contact angles of condensate in air (\(\theta_{wo(a)}\)) and oil (\(\theta_{wo(o)}\)), respectively, and lie in the limits of \(0 < \psi_1, \psi_2 < 1\). As a result even if the interfacial tension between a condensate and a liquid is less than the surface tension of condensate in air (i.e. \(\gamma_{wo}/\gamma_{wa} < 1\)), \(E_{III}/E_I\) can be greater than one so that nucleation in the air environment may be more preferred compared to nucleation within a liquid, contrary to the hypothesis of Xiao et al. At the same time, even if \(\gamma_{wo}/\gamma_{wa} > 1\), nucleation within the liquid may be enhanced if the contact angle terms are such that eqn (2) is satisfied. The regime map satisfying eqn (2) is shown in Fig. 1b where the marked regions are the conditions under which nucleation in state III is more favourable than state I. A decrease in ratio of \(\gamma_{wo}/\gamma_{wa}\) can drastically decrease the actual energy barrier so that even non-wetting surfaces in an oil have smaller \(SR^*\) of nucleation compared to wetting surfaces in air (see ESI Fig. S2†). Substituting \(\theta_{wo(a)} = 180^\circ\) and \(\theta_{wo(o)} = 90^\circ\) for water in eqn (2), we find that \(E_{III}/E_I < 1\) for all oils with \(\gamma_{wo}/\gamma_{wa} < 0.79\) (a condition met by most common oils with respect to water – see ESI Table 1† for examples), suggesting that homogeneous nucleation and thus by extension heterogeneous nucleation on any solid surface within such oils is favored than nucleation on a non-wetting surface in the air.
For the case when the oil does not cloak the condensate i.e. \( S_{\text{ow(a)}} < 0 \), and, the condensate does not wet the oil i.e. \( S_{\text{ow(a)}} = \gamma_{\text{oa}} - \gamma_{\text{wa}} - \gamma_{\text{wo}} < 0 \), nucleation at the oil–air interface (State IV) is preferable over nucleation at the solid–air interface (state I) if

\[
\frac{E_I}{E_{IV}} = \frac{\ln(SR_i^I)}{\ln(SR_{IV}^I)} = \frac{(\psi_i^2)^{1/3}}{\xi} \sin \theta_{wa} > 1
\]

where

\[
\xi = \left( \frac{1}{1 + \cos \theta_{wo} - \cos \theta_{wo}} \right) + \frac{\sin \theta_{wo} \sin \theta_{wa} (2 + \cos \theta_{wo})}{(1 + \cos \theta_{wo})^2} \frac{2}{\sin \theta_{wo} - \cos \theta_{wa}}
\]

and

\[
\lambda = \frac{\sin \theta_{wo} (2 + \cos \theta_{wo})}{(1 + \cos \theta_{wo})^2} + \frac{\sin \theta_{wa} (2 + \cos \theta_{wa})}{(1 + \cos \theta_{wa})^2}
\]

(see ESI Note 1†).

Similarly, nucleation at the oil–air interface (state IV) is preferable over nucleation at the solid–oil interface (State III) if

\[
\frac{E_{III}}{E_{IV}} = \frac{\ln(SR_{III}^I)}{\ln(SR_{IV}^I)} = \frac{(\psi_i^3)^{1/3}}{\xi} > 1
\]

In state IV, the two lens angles \( \theta_{wa} \) and \( \theta_{wo} \), are defined with respect to the plane of fluid and are bounded by \( \theta_{wo} + \theta_{wa} \leq 180^\circ \). Here we consider the two cases that correlate the interfacial tensions at the contact line with the lens shape. For the first case, we consider oils with \( \gamma_{\text{ow}}/\gamma_{\text{wa}} < 1 \). Since \( \gamma_{\text{ow}} \sin \theta_{wo} = \gamma_{\text{wa}} \sin \theta_{wa} \) from the force balance at the three phase contact line, this implies that for such oils \( \theta_{wa} < \theta_{wo} \). Combined with the criterion \( \theta_{wa} + \theta_{wo} \leq 180^\circ \), this shows that for all oils with \( \gamma_{\text{ow}}/\gamma_{\text{wa}} < 1 \), the lens contact angle \( \theta_{wo} \) is bound by \( \theta_{wa} \leq \theta_{wo} \leq 180^\circ - \theta_{wa} \) and \( \max(\theta_{wa}, \theta_{wo}) = 90^\circ \). Similarly, considering the second case when \( \gamma_{\text{ow}}/\gamma_{\text{wa}} \geq 1 \), the lens contact angle \( \theta_{wa} \) is bound by \( \theta_{wo} \leq \theta_{wa} \leq 180^\circ - \theta_{wo} \). The regime maps satisfying eqn (3) and (4) are shown in Fig. 1c and d respectively. Also shown are regions corresponding to \( \gamma_{\text{ow}} < \gamma_{\text{wa}} \) (\( \theta_{wa} < \theta_{wo} \) blue), \( \gamma_{\text{ow}} > \gamma_{\text{wa}} \) (\( \theta_{wo} < \theta_{wa} \) green), and no-nucleation (\( \theta_{wo} + \theta_{wa} > 180^\circ \) grey). Clearly, condensation at the oil-air interface is always favorable when compared to nucleation on a perfectly non-wetting solid \( \theta_{wa(o)} = 180^\circ \) or \( \theta_{wo(a)} = 180^\circ \) regardless of the environment. Fig. 1c shows that compared to hydrophobic surfaces in air, oils in which droplets remain largely immersed (i.e. oils with \( \gamma_{\text{ow}} < \gamma_{\text{wa}} \)) have lower energy barrier. The extent of such a lowering can even allow droplets to nucleate at significantly low supersaturation when compared to wetting solid surfaces in air (see ESI Fig. S3†). For oils with \( \gamma_{\text{wo}} > \gamma_{\text{wa}} \), the number of combinations of \( \theta_{wo} + \theta_{wa} \) that allow for nucleation enhancement are greatly restricted, mainly the oils with \( \theta_{wa} < \theta_{wo(a)} \). On the other hand Fig. 1d shows that for oils with \( \gamma_{\text{wo}} > \gamma_{\text{wa}} \), droplets that are largely immersed in air can nucleate more readily compared to nucleation within the oil. For such oils, nucleation can occur at very low supersaturations (see ESI Fig. S4†). In general, for oils with \( \gamma_{\text{wo}} < \gamma_{\text{wa}} \), nucleation in state III is more favorable when compared to state IV if the condensate lens angle \( \theta_{wo} > \theta_{wo(a)} \).

The preceding analysis is based on the assumption that the oil-air interface is atomistically smooth. However, random thermal fluctuations can induce thermal-capillary waves, whose mean amplitude is expected to be on the order of \( (4\pi kT/\gamma)^{1/2} \approx 10–20 \) Å for low surface tension oils \( \gamma < 30 \) mN m\(^{-1} \). Studies have shown that thermal capillary waves play an important role in the coalescence of droplets and spreading of liquids. The work of cluster formation at the oil–air interface could be lower than our estimate if we consider the dynamic roughness induced by such thermal-capillary waves; detailed study of these effects needs to be conducted and is out of scope of the present paper.

### 3.2 Nucleation dynamics in an immiscible oil

The framework for nucleation energy barrier developed in the preceding section can be used to determine the most preferred state for nucleation in different environments (oil or air). As an example, we find that on a LIS with hydrophobic solid surface, the energy barrier for nucleation of water in the presence of many oils is the highest in State I, and the lowest in State IV (see ESI Table 2 and Fig. S2–S4†). At the same time, as Fig. 1c and d show, nucleation within the oil (State II or III) can be favourable for many oil–solid combinations. However, a low nucleation energy barrier may not correspond to large nucleation rates, since the nucleation rate also depends on other factors such as nucleation site density, the diffusion coefficient of molecules and the sticking probability of molecules (also referred to as the accommodation coefficient) in an environment. One may expect that very large supersaturation would be required in oils such as silicone oils or fluoropolymers (e.g. Krytox) that have low water miscibility to produce nucleation rates of similar magnitude as observed in air (see ESI Table 3 for a list of water solubilities in immiscible oils). Surprisingly, we find that under identical supersaturation, and assuming the accommodation coefficient as one, the nucleation rates within these oils can be of similar magnitude as the nucleation rates that can be obtained within air (see ESI Note 2†). The latter assumption however may be excessive for these oils. MD simulations show that the accommodation coefficient of water molecules on a water droplet covered with a monolayer of long chain organic alcohols could be substantially lower than one. If the behaviour of water molecules in Silicone oils or Krytox is similar to their behaviour in the long chain organic liquids, then much higher supersaturation may indeed be required to nucleate a large number of droplets in these oils (see ESI Note 2†).

However, a more fundamental question to ask is can the vapor species become supersaturated in a liquid that is in contact with the condensing vapor upon subcooling.

The transport of a gaseous species through the liquid occurs by the sorption and diffusion mechanism. According to Henry's Law, the maximum volume of vapor \( C_v \), absorbed in a unit volume of liquid is given by \( C_v = H_s P_v \) where \( H_s \) is Henry's constant of solubility of vapor in the liquid at a given temperature \( T_v \) and \( P_v \) is the partial pressure of vapor above the liquid held at the same temperature as air. Since the dissolution of gas in a liquid is an exothermic process, the solubility limit of
vapor in liquids (and the Henry’s Constant) is expected to increase when the temperature is decreased.\textsuperscript{24,75} When a liquid is cooled to a temperature \( T_i \) (\(<\)the room temperature), then as long as condensation does not occur in air region near the subcooled liquid, the partial pressure of vapor near the liquid–air interface remains unaffected. Since the total pressure remains the same (equal to absolute pressure), the maximum amount of vapor that can get absorbed is \( C_{i,j} = H_i P_v \), where \( H_i (\geq H_c) \) is the Henry’s constant at temperature \( T_i \). Thus, as the liquid is cooled, it becomes under-saturated. The liquid absorbs more vapor due to increased solubility but only till its new solubility limit. Consequently, the vapour cannot supersaturate within the liquid and droplet formation by condensation cannot occur.

To validate this aspect, we conducted a series of experiments. In the first experiment, we lowered the temperature of a liquid below the room-temperature to ascertain if condensation occurs within the oil [see Methods]. To prevent supersaturation in the air near the setup, the temperature was maintained above the dew point in air (Peltier temperature: \(16 \pm 1\)°C, Dew Point: \(13 \pm 1\)°C). Silicone oil of viscosity 10 cSt was chosen as a test liquid because of its low vapor pressure. Despite exposing the oil to the humid environment (room humidity of 47\%) for two hours, no trace of condensation was observed within the oil (Fig. 2a). Condensation however proceeded immediately when the temperature was lowered below the dew point in air (Fig. 2b) and microscopic droplets were identified at the optical plane near the oil–air interface.

In the second experiment, we performed a more rigorous test to determine if water drops can condense within the oil in the complete absence of the oil–air interface by lowering its temperature substantially. A 10 cc glass vial was completely filled with a solution of 10 cSt silicone oil saturated with moisture (see Methods and ESI Fig. S1† for the schematic of the setup). The vial was wrapped with insulation and cooled to \(-2\)°C using a peltier cooler for a period of three hours. To detect the formation of nanoscale drop formation within the oil, we performed dynamic light scattering (DLS) measurements on the solution samples extracted from the vial within 30 minutes of taking the vial off the peltier cooler. The DLS instrument used in the study had a minimum detection size of 0.2 nm in hydrodynamic radius. We performed multiple measurements over different volumes of the solution, however the DLS measurements showed a complete absence of any droplet formation within the oil.

Based on the above results, it is unlikely that nucleation can occur within the bulk oil. Our observations suggest that the formation of droplets on the bulk silicone oil is directly linked with the saturation dynamics in air. In the case of a subcooled oil exposed to air, the region of supersaturation lies in the air beyond the oil–air interface and hence nucleation of a droplet is likely to occur at the oil–air interface, irrespective of the nature of oil.

### 3.3 State of the droplet on spreading oils after-nucleation: cloaked or uncloaked?

Having established that the nucleation is most likely to occur at the oil–air interface, we now investigate the dynamics during the droplet growth process, especially on oils that cloak the condensates. Post-nucleation, the growth of a droplet at the oil–air interface on a non-cloaking oil can readily occur through direct diffusion of the vapor to the drop surface.\textsuperscript{40} For droplets growing at the oil-air interface on spreading oils, a layer of the oil might be introduced between the vapor and the drop if the rate of droplet growth (\(U_d\)) is lower than the spreading rate of oil on the condensing droplet (\(U_s\)). In the opposite case, the drop surface will be exposed and the condensation on spreading oil can be expected to behave similar to condensation on non-spreading oil or a solid surface.

The growth of droplets is influenced by several factors such as droplet density, saturation conditions etc.\textsuperscript{1,43,58} To estimate droplet growth rate (\(U_d\)), we consider the growth of an isolated droplet at the oil–air interface. We assume that the droplet at the oil–air interface has a lens shape, and the droplet and the oil are at temperature \( T_i \). With these assumptions, the droplet growth law is given by \( R = \eta^{1/3} \sqrt{2\pi T_i} \) where \( R \) is the radius of curvature of the upper segment of the lens, \( \eta \) is a geometric factor that relates the volume change of the lens with condensation at the lens-air interface. The detailed derivation of our model is provided in the ESI Note 3.\textsuperscript{†} The droplet growth rate (\(U_d = dR/dt\)) is given by

\[
U_d = \phi^{1/3} \sqrt{\frac{\eta}{27}}
\]

(5)

where

\[
\eta = \frac{4 M_w D_{ab} P_o SR_0}{\rho_v \psi_{wa} RT_i} \left( \frac{T_i}{T_v} - 1 \right) SR \quad \text{and} \quad \phi^{-1} = 1 + \frac{\psi_{wa}}{\psi_{oa}} \left( \frac{\sin \theta_{oa}}{\sin \theta_{wa}} \right)^3
\]

Here \( M_w, \rho_v, D_{ab}, SR, T_i, T_v, P_o, R \) denote the molecular weight of the condensate, density of the condensate, diffusion coefficient of vapor in air, saturation ratio, vapor temperature, droplet temperature, saturation vapor pressure at temperature \( T_i \), and gas constant, respectively. \( \psi_{wa} = (2 + \cos \theta_{wa})(1 - \cos \theta_{oa})^2 \) and \( \psi_{oa} = (2 + \cos \theta_{oa})(1 - \cos \theta_{wa})^2 \) are the shape factors of the lower and upper segment of the lens (Fig. 1a, State
IV). Having estimated the growth rate of droplets, we now consider the spreading rate of the oil on the droplet. The spreading of oil on a drop can be delineated into two stages. During the first stage, a monolayer driven by balance between surface tension gradient and shear stress at the oil-droplet interface spreads on the droplet. For the radial spreading of an oil monolayer on water, it has been shown that the spreading front location follows Joos law and is given by:

$$R_{s,lm} = \sqrt[3]{(2\pi \rho_o S_{ow}(a)/3\sqrt{\mu_o \rho_o})^{1/3}}$$

from where the spreading velocity is found as

$$U_{s,lm} = \sqrt{3S_{ow}(a)/4\sqrt{\mu_o \rho_o}^{1/3}}.$$  

Here $\rho_o$, $\mu_o$ denote the density and dynamic viscosity of the oil and $S_{ow}(a)$ is the spreading coefficient of oil on water as measured in air. Although the spreading of oil around a droplet is expected to be greater compared to spreading of oil in a plane, the latter can be used as an approximation for the spreading velocity around a droplet. The monolayer is followed by a nanofilm with thickness up to few hundred nanometers. The spreading rate of this nanofilm is dictated by the capillary forces opposed either by inertial or viscous forces. To determine the predominant dissipating force during this spreading regime, we consider the Ohnesorge number ($\text{Oh} = \mu_o/\sqrt{\rho_o R_{s,lm}^3}$) of the oil with the characteristic length of the droplet radius. For water droplets of size $<100$ nm on Silicone oil of viscosity $10$ cSt and above, we find that $\text{Oh} > 1$, implying that the spreading of oil on droplets during growth occurs in the viscous regime. In a recent work, Carlson et al. have shown that for an oil spreading on a droplet in the viscous regime, the spreading front location follows $R_{s,lm} = 0.87R_{(\text{lm})}^{0.3}$ from where the spreading velocity on a growing droplet is given by:

$$U_{s,lm} = \frac{0.72\pi^{0.35}(\text{lm})^{0.3}T_{\text{lm}}^{-0.35}}{\text{lm}}.$$  

Knowing the droplet growth rate and cloaking rates of the monolayer and nanofilm, we can now establish if droplets remain cloaked or not during the growth process. Comparing the droplet growth rate with the monolayer cloaking rate, we find:

$$U_{s,lm}/U_d \sim k_m^{0.25}$$

where

$$k_m = \sqrt{3S_{ow}(a)/2\rho_o^{3/2} \eta\mu_o \rho_o}.$$  

Depending upon the value of the pre-factor $k_m$, the droplet may remain uncloaked for a time $t_{\text{uncloak}}$ during which $U_{s,lm}/U_d \sim k_m^{0.25} < 1$. For a water nanodroplet on silicone oil of viscosity $1000$ cSt, substituting the relevant values ($S_{ow}(a) = 5$ mN m$^{-1}$, $\rho_o = 970$ kg m$^{-3}$, $\rho_w = 980$ kg m$^{-3}$, $T_o = 293 K$, and $D_{ab} = 2 \times 10^{-5}$ m$^2$ s$^{-1}$) we find that depending upon the lens angles ($0 < \theta_{wa} < 180^\circ$ and $\theta_{wo} \leq 180^\circ$ since the $\theta_{wa}$, $\theta_{wo}$ are unknown) and supersaturation ($\text{SR}$), the constant $k_m \sim 10^{1-10^4}$ from which the $t_{\text{uncloak}}$ can be estimated to be between $10^{-18}$ to $10^{-4}$ s. The large values of $t_{\text{uncloak}}$ are obtained for large supersaturation ($T_o - T_i = 20 K$) and for very small lens angles ($\theta_{wa}, \theta_{wo} < 10^\circ$).

Next, comparing the droplet growth rate with the spreading rate of the nanofilm, we find $U_{s,lm}/U_d \sim k_d^{0.15}$ where

$$k_d = 1.02\rho_o^{0.15}(\text{lm})^{2/3}\pi^{2/3}\mu_o^{0.15}.$$  

Since $U_{s,lm}/U_d \sim l_{\text{lm}}^{0.15}$, the spreading of nanofilm will overcome the droplet growth eventually. By substituting the relevant values for the water nanodroplet on Silicone oil we find that depending upon the lens angles and supersaturation ratios, the time taken by the thicker sub-microscopic film to form around the growing droplet lies in the range of $10^{-9} - 10^{-3}$ s. Based on above calculations, we conclude that after a tiny fraction of time, the condensing droplets on a cloaking oil are cloaked by the oil nanofilm.

Although, visualization of nanodroplet formation and the cloaking process during the initial droplet growth is challenging, the evidence for the last statement can be obtained by indirect experimental observations. An oil film cloaking a droplet would tend to submerge the droplet within the oil in order to minimize its own surface energy. As a consequence, condensed droplets are expected to be located in the oil in a fully submerged state. Results from the optical microscopy (Fig. 2b) indicate the presence of droplets near the oil–air interface; however, the exact location of the droplets could not be determined because of limits in spatial resolution of the microscope. Recently, Rykaczewski et al. developed a technique that can reveal nanoscale details of the underlying structure of droplets and substrate interfaces. Using this technique, we obtained direct cross-sectional images of the topography beneath the $1000$ cSt and $10$ cSt silicone oil surfaces up to a depth of $10-20$ μm, after condensing vapor on them under the same conditions (see Methods). The regions on the oil surfaces for obtaining the cross-sectional images were selected randomly. By visualizing the Cryo-SEM images using the back-scattered detector, the water and oil phases were separately identified (see Methods for an extended description on phase identification). Images in Fig. 3a-d show the morphology of $1000$ cSt and $10$ cSt silicone oil surfaces with condensed
droplets before and after sectioning. The cross-sectional images of the 1000 cSt and 10 cSt silicone oil surfaces (Fig. 3b and d) clearly show the presence of fully submerged droplets (dark grey in color) within the oils (light grey in color), thereby confirming the previously predicted behaviour of droplets condensing on the cloaking liquid.

From the images obtained using Cryo-FIB-SEM (Fig. 3), several other insights into the droplet growth mechanics can be drawn. Fig. 3a shows that prior to the sectioning of the 1000 cSt Silicone oil surface, the surface appeared rough with microscale features emerging out of the oil. The cross-section of the 1000 cSt oil surface (Fig. 3b) shows the presence of uneven size droplets arranged in stacks within the oil. The stacking of the droplets leads to deformation of the oil–air interface, giving rise to the appearance of roughness observed in Fig. 3a. The droplets also appear to be densely packed, with less than 100 nm separations between many neighboring droplets (see Fig. S3–S6† for more examples). In comparison with the condensation pattern on the 1000 cSt silicone oil surface, the oil–air interface of the 10 cSt silicone oil surface appeared to be relatively smoother (Fig. 3c). The cross-sectional image of the selected region showed the presence of a single fully submerged droplet within the oil (Fig. 3d). The apparent difference in the submerged droplet sizes in similar volumes of the two oils is attributed to the different viscosities of the oils that affect the coalescence behaviour of the droplets.

Based on the results and arguments mentioned above, the nucleation and submergence mechanism on cloaking oils is proposed to occur in the following steps (Fig. 4): (a) a droplet nucleates at the oil–air interface, (b) subsequently, the droplet is cloaked by the oil, (c) the cloaking leads to submergence of the droplet within the oil due to capillary forces, thereby creating a fresh oil–air interface, and (d) finally, the cycle (a)-(c) is repeated with new generation of droplets forming at the oil–air interface and submerging. The interaction between the old and the new generation of droplets may lead to re-organization of droplets. Depending upon the oil viscosity, it may result in different arrangements within the oil (such as stacked arrangements as observed in Fig. 3b, or a single droplet Fig. 3d). A more detailed examination of this aspect will be performed at the later part of this work.

The preceding discussion relates to the fate of droplets whose size is smaller than the oil thickness surrounding them. But even if the droplet size becomes larger than the oil thickness surrounding it, the droplet still remains cloaked as it grows. The local equilibrium thickness of the cloak profile around such droplets is dependent upon the balance between spreading forces (due to repulsive Van der Waal’s interaction between oil and vapor) that tend to thicken the cloak around the droplet, and the positive pressure gradient developed in the film due to difference in disjoining pressure and the hydrodynamic pressure that tends to thin down the cloak. Formation of the cloaked film on a droplet can occur with a minimal contact between the spreading oil and a droplet, e.g. for a droplet suspended on a LIS. To directly visualize the presence of such a cloaked film, we used the cryo-FIB-SEM technique to obtain a cross-section of a randomly selected condensed droplet with size larger than the post-spacing on 10 cSt Silicone oil LIS (Fig. 5). Although the oil cloak thickness may be a function of time, oil properties, droplet size etc., Fig. 5 provides a general representation of the cloak profile around the droplet. The images show that the cloak profile around the droplet decreases sharply beyond the wetting ridge height. The thickness profile was estimated at ~65 nm around the droplet and remains mostly uniform around the droplet. Surprisingly, we find a thicker oil profile near the apex of the droplet. A closer inspection of this region shows the presence of two sub-microscopic droplets with a diameter of ~100 nm and 250 nm within the oil film (a magnified image on top of Fig. 5, and ESI Fig. S11† provided separately). It is likely that these droplets nucleated on the oil cloak and the tendency of the oil to form the cloak around these droplets provided the driving force to cause oil imbibition that resulted in the thickening of the oil cloak.

As evident from the observations of Fig. 2 and 5 and condensation on cloaking liquids in prior studies, the growth of nanoscopic droplets to larger sizes occurs despite the complete engulfment of the droplets by the oil. In the next sections, we discuss the mechanisms of droplet growth on the cloaking liquids.

### 3.4 Droplet growth: the role of permeation

It is well known that the growth of droplets on surfaces occurs via either direct vapor accretion at the droplet surface, or via coalescence with neighbouring droplets. For droplet growth on a cloaking liquid, the first mechanism is improbable because the oil film acts as a barrier against direct diffusion of vapor molecules to the droplet surface. Despite the oil layer acting as the barrier, vapor molecules can permeate through the film in the presence of concentration gradient across the film.

However, the role of permeation and diffusion in the growth of a droplet submerged within the oil is unclear. A droplet of radius $R$ immersed within the oil has excess pressure due to its curvature, and as a result, the chemical potential of the
dispersed phase at the droplet surface and in the bulk are different. From the Kelvin equation, the concentration of molecules (\(C_l\)) around a droplet is given by \(C_l = \frac{C_s \exp[(2\gamma_{wl})/(\rho_w R T)]}{\rho_w}\). Here, \(C_s\) is the bulk phase solubility and \(\rho_w\) is the density of the dispersed phase (water). Thus the concentration of dispersed phase molecules around the droplet is higher than the concentration within the oil. For droplet growth to occur within the oil, the solute (vapor here) content within the oil must exceed \(C_r\). However as described in the preceding section, the vapor saturation in the oil is limited by the sorption mechanism and this makes it unlikely for vapor to achieve supersaturation in the oil. Even if the oil layer thickness is sub-microscopic, the solute transport across the film is governed by the sorption mechanism (e.g., in studies on coarsening of the foams, the gas permeation across the thin lamellae is described using this mechanism, see ESI Note 3† for more discussion on this aspect). Based on above arguments, droplet growth through permeation of vapor molecules in the oil appears unlikely.

Despite the limits on the vapor content within oil due to sorption, vapor supersaturation within the oil may be possible via other mechanisms. As an example, the presence of nucleated droplets at the oil–air interface can alter the solute content within the oil. In the previous paragraph it was explained that because of the droplets’ curvature, the droplet surface has excess solute concentration compared to the bulk solubility limit. If the oil is under-saturated, then the droplet dissolves with diffusion within the oil acting as the rate-limiting step. In general, this mechanism could result in the increase of supersaturation within the oil that may result in heterogeneous or homogeneous nucleation within the oil or act as the source of growth of other droplets. However, identifying the contribution of vapor diffusion within the oil to the overall growth of a submerged droplet is difficult because of several reasons. First, the nucleation rate at the oil–air interface is difficult to estimate precisely. Secondly, the percentage and size of the nucleated droplets that may dissolve is unknown. As a result, it is challenging to estimate the supersaturation within the oil layer due to droplet dissolution.

### 3.5 Droplet growth: the role of oil viscosity and coalescence

The difference in droplet sizes is observed through cryo-FIB-SEM (Fig. 3a–d) and the presence of droplets within the oil cloak (Fig. 5) is attributed to the presence of the intervening oil film around the droplets, and its effect on delaying coalescence. The process of coalescence in the presence of an outer viscous fluid occurs in two stages. During the first stage, the oil drains from in-between two droplets thereby bringing them into sufficient proximity so that small instability propagating on the droplet surface can cause a capillary bridge formation. Once the two droplets are in contact, the bridge expands to achieve a minimal energy state.

The dynamics of coalescence between two neighbouring droplets at an interface during condensation is a complex function of their size, growth rate of droplets, and attractive forces due to capillary interactions between them. Solving for...
the complete drainage between condensing droplets is beyond the scope of this work. However using the Stefan Reynolds Flat plate model,\textsuperscript{64,66} we find that when the droplets are separated by distances where the drainage happens purely to van der Waals forces, the drainage time is directly proportional to oil viscosity (see ESI Note 4†). The delay in coalescence of macroscopic droplets placed in each others vicinity (~mm) on LIS due to decreased drainage rates in the case of higher viscosity oil has recently been also confirmed.\textsuperscript{67}

We thus expect the viscosity of the oil to have a significant effect on growth of droplets during condensation. Previous studies on breath-figure formation on polymer–solvent mixtures\textsuperscript{68} have also hinted towards its importance, however the use of solvent alters the solution viscosity and masks the true effect of the oil viscosity on condensation. To observe this effect, we performed condensation experiments on LIS prepared by impregnating OTS coated microtextured surface with silicone oils of viscosity of 10, 100 and 1000 cSt (see Methods). The impregnation of the OTS coated surface with silicone oils results in complete submergence of solid (including post-tops) in the presence of air and water.\textsuperscript{29} Upon condensation, we notice the formation of darkened regions (as a result of droplets nucleating on the post-tops) and their subsequent disappearance (owing to their getting pulled within the oil spacing due to capillary forces originating from the Laplace pressure of the oil cloak around the droplet). As a result of submergence of such droplets and of droplets nucleating on oil itself, the water droplets displace the oil resulting in the oil draining out of the LIS and flooding the surface. However, as the size of the submerged droplets exceeds the post-spacing, they can transition to the post-tops and the oil can flow back within the texture to fill in the void left behind. For low viscosity oil cases (10 cSt and 100 cSt LIS) the oil displacement appears less severe compared to the 1000 cSt LIS because in the former cases, the oil can drain quickly between the submerged droplets allowing them to coalesce more rapidly and grow at a faster rate. On 1000 cSt LIS, significant suppression of condensation growth is observed that we postulate is due to the increased drainage time and higher oil content around the droplets caused by the oil displaced from within the texture (Fig. 6a–c, also see ESI Movie

**Fig. 6** Effect of liquid viscosity during condensation on LIS. (a) Time sequence showing growth of condensed droplets as observed under a microscope on the micropost surface (identical surfaces are used in Fig. 5) impregnated with Silicone oil of viscosity (a) 10 cSt, (b) 100 cSt and (c) 1000 cSt. The experiments were performed in an open environment under the same conditions (Peltier temperature = 3 ± 1°C and dew point = 12 ± 1°C). Even on 100 cSt, significant resistance to coalescence is observed. On the 1000 cSt Silicone oil surface, there is significant inhibition against coalescence and condensed droplets are separated through a thin oil film that takes orders of magnitude larger time to collapse as compared to droplets cloaked with 10 and 100 cSt viscosity silicone oil. (d) Plot comparing the variation of the droplet occupied area fraction versus time on 10, 100 and 1000 cSt silicone oil impregnated surfaces. On the 10 cSt surface, the droplet coverage reaches ~55% as is normally observed on condensation on solid surfaces. On the 100 cSt surface, there is an initial delay in forming of large droplets within the observed frame, but large size droplets are formed and move out of frame due to coalescence events. In comparison, the droplet coverage reaches ~90% within minutes on the 1000 cSt surface. (e) Plot comparing the variation of droplet sizes (polydispersity) versus time on 10, 100 and 1000 cSt Silicone oil impregnated surfaces. The actual polydispersity and area coverage on 100 cSt was significantly higher, but the spatial resolution limits prohibited identification of individual droplets from the background.
To quantify the difference in growth behaviour, we obtained droplet coverage over time (Fig. 6d) and the polydispersity in size distribution over these surfaces (Fig. 6e, see Methods). From Fig. 6d, it is evident that the fraction of surface area occupied by the droplets increases continuously across all the three samples. On 10 cSt LIS, the droplet area fraction rapidly reaches a coverage close to $50\%$–$55\%$, similar to the average area fraction observed during condensation on solid surfaces. Although the droplets are cloaked, but the drainage of oil between the droplets is more efficient due to which the drops can coalesce rapidly, thus leaving a large fraction of the surface unoccupied by the droplets. This is also evident from the polydispersity graph (Fig. 6e), where it can be seen that droplet sizes on 10 cSt LIS become increasingly polydisperse with time. The image analysis of 100 cSt LIS was less accurate because the droplet growth behavior on this surface made it difficult to identify the droplet boundaries. Although the area coverage on 100 cSt LIS shows similar trends as observed on 10 cSt LIS, the actual coverage was larger.

On the 1000 cSt surface, a fascinating range of droplet growth behavior with several distinctive features was observed. First, after a short duration, the polydispersity in size distribution vanished and a very narrow size-distribution of droplets was obtained. Second, significant inhibition against coalescence was observed; yet the droplet size increased with the passage of time evidenced by the continuous increase in droplet coverage over the surface (Fig. 6d). Third, the droplet shape changed from spherical to polyhedral with time and condensed droplets appear to self-assemble in closely packed honeycomb-like structures (Fig. 6c). Initially, the condensation pattern was reminiscent of wet foam architecture, while at later times the condensation pattern resembled dry foam architecture. The polyhedral droplet profiles are separated through thin films resembling plateau borders and intersect at $\sim120^\circ$ as dictated by the equilibrium requirement for three equal surface tension forces at intersection$^9$ (also see ESI Fig. S7†). Finally, the focal plane of the microscope constantly needed to be raised to keep a sharp focus on the droplets, implying that multiple layers of stacked droplets were being formed.

4. Conclusions

In summary, we have examined the detailed mechanics underlying the droplet formation and subsequent growth processes on liquids. By examining at the energetics of the nucleation process, we have provided regime maps that can guide the selection of liquids that enhance nucleation rates at lower supersaturation. Here we find that the vapor content within the liquid is limited by the sorption mechanisms, so that nucleation on subcooled liquid is most likely to occur at the liquid–air interface. By investigating the spreading rates of cloaking liquids and the condensate growth rate, we have provided a mechanistic understanding of the process that leads to submergence of droplets that nucleate at the liquid–air interface. While the submergence of condensing droplets within the cloaking liquids maybe useful in some applications (such as breath-figure templating); for the application of LIS in condensation, it can potentially decrease the longevity of the coating by displacing the impregnated liquid out of the texture. For the first time, we provide a direct visualization of the nanoscale liquid film profile around a droplet and show that sub-microscopic droplets can nucleate on the liquid cloak itself. Finally, by doing a systematic study of condensation on different viscosity oils, we have uncovered the role of oil viscosity in governing the droplet growth behaviour.

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References


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Supporting Information: How droplets nucleate and grow on liquids and liquid impregnated surfaces

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KEYWORDS liquids, liquid-impregnated, dropwise condensation, breath-figures, nucleation, growth, coalescence
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* Measured ** Calculated from \(\cos \theta_{wo} = \frac{\gamma_{oa}^2 + \gamma_{wo}^2 - \gamma_{wa}^2}{2\gamma_{oa}\gamma_{wo}}\) \(\text{and } \cos \theta_{wa} = \frac{\gamma_{oa}^2 + \gamma_{wo}^2 - \gamma_{ow}^2}{2\gamma_{oa}\gamma_{wo}}\)

** Supplementary Table 1: ** Spreading Coefficients of some common liquids on water. The thermodynamic properties (density, surface tension) correspond to measurements at room temperature \((T=20\ ^\circ\text{C})\)
### Supplementary Table 2: Comparison of interfacial energy of cluster formation and critical saturation ratios

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<td>1.328</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-Bromonaphthalene</td>
<td>0.62</td>
<td>0.653</td>
<td>0.624</td>
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<td></td>
<td></td>
<td>1.267</td>
<td>(\theta_{ws(a)}=160^\circ)^*</td>
<td>1.254</td>
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<tr>
<td>[BMIm$^+$][Tf2N$^-$] (final)</td>
<td>0.31</td>
<td>0.202</td>
<td>0.194</td>
<td>0.962</td>
<td></td>
<td></td>
<td>1.077</td>
<td>(\theta_{ws(a)}=143^\circ)^*</td>
<td>1.073</td>
<td></td>
<td></td>
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</tbody>
</table>

* Measured.

\(SR^* = \exp\left(\frac{2\beta}{(3kTn^{1/3})}\right)\) for nucleation of 2 nm diameter droplets in different states on a LIS with hydrophobic solid surface for some common liquids. The hydrophobic surface here is an OTS (octadecyltrichlorosilane) coated smooth silicon surface. The contact angles (\(\theta\)) of the liquids were measured within water using goniometer. From these measurements, the contact angle of water within the liquid was evaluated as \(180^\circ - \theta\) where \(\theta\) denotes value corresponding to the average of advancing and receding contact angles of droplets on the surface.
<table>
<thead>
<tr>
<th>Liquid</th>
<th>Density ($\rho_o$)</th>
<th>Solubility of Water in Liquid</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g/cc</td>
<td>(ppm by weight)</td>
</tr>
<tr>
<td>1,2-dibromotetrafluoroethane</td>
<td>2.162</td>
<td>$3^{(7)}$</td>
</tr>
<tr>
<td>Krytox</td>
<td>$1.86^{(6)}$</td>
<td>$17^{(8)}$</td>
</tr>
<tr>
<td>Octadecane</td>
<td>$0.765^{(1)}$</td>
<td>$50.33^{(7)}$</td>
</tr>
<tr>
<td>Hexadecane</td>
<td>$0.77^{(1)}$</td>
<td>$53.97^{(7)}$</td>
</tr>
<tr>
<td>Tetradecane</td>
<td>$0.76^{(1)}$</td>
<td>$57.92^{(7)}$</td>
</tr>
<tr>
<td>Toluene</td>
<td>$0.9^{(1)}$</td>
<td>$541.78^{(7)}$</td>
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<tr>
<td>Silicone Oil</td>
<td>0.94-0.97</td>
<td>$700^{(9)}$</td>
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<tr>
<td>Chloroform</td>
<td>$1.5^{(1)}$</td>
<td>$871^{(7)}$</td>
</tr>
<tr>
<td>Dichloromethane</td>
<td>$1.3^{(1)}$</td>
<td>$1761.6^{(7)}$</td>
</tr>
</tbody>
</table>

* $C(\text{mol/m}^3)=10^{-3} \times [\rho \,(\text{g/cc})\times 1000] / [M_{\text{water}}(\text{g/mol})] \times C(\text{ppm})$ where $M_{\text{water}}=18$

* These solubilities correspond to STP conditions.

**Supplementary Table 3:** Water solubilities in some liquids
**Supplementary Figure S1:** Schematic of Experiments. (a) Setup for condensation experiments (b) Setup for generating vapor saturated oil.
Supplementary Figure S2: Energy Barrier between State I and State III. Critical saturation ratio for nucleation at 20 °C of 2 nm diameter droplets for variable contact angles of water on the solid surface in presence of air (State I, blue curve) and oil (State III, orange curves) as a function of surface wettability in air ($\theta_{ws(a)}$), in oil ($\theta_{ws(o)}$), and $\gamma_{wo}/\gamma_{wa}$. Also shown on the plot are critical saturation ratios for nucleation of water on a OTS coated surface in air, and on OTS coated surface immersed in different oils. It can be seen $SR^*$ for nucleation on OTS is highest in air environment.
Supplementary Figure S3: Energy Barrier between State I and State IV. Critical saturation ratio for nucleation at 20 °C of 2 nm diameter droplets on the solid surface in presence of air (State I, blue curve) and at oil-air interface (State IV, orange curves) as a function of contact angle of droplet in air ($\theta_{wa(a)}$), lens angles of droplet ($\theta_{wo}$, $\theta_{wa}$). Here solid orange lines represent oils with $\gamma_{wo} \leq \gamma_{wa}$ and dashed orange lines represent oils with $\gamma_{wo} \geq \gamma_{wa}$. Also shown on the plot are $SR^*$ for nucleation of water on a OTS coated surface in air, and $SR^*$ for nucleation at oil-air interface for some liquids. It can be seen $SR^*$ for nucleation is highest on the OTS coated solid surface.
Supplementary Figure S4: Energy Barrier between State III and State IV. Critical supersaturation for nucleation at 20 °C of 2 nm diameter droplets within the oil (State III, blue curve) and at oil air interface (State IV, orange curves). However instead of $SR^*$, we plotted $(SR^*)^{1/\gamma_{w0}}$ to obtain the graph independent of the oil-droplet interfacial tension. Here solid orange lines represent oils with $\gamma_{w0} \leq \gamma_{wa}$ and dashed orange lines represent oils with $\gamma_{w0} \geq \gamma_{wa}$. Also shown on the plot are $SR^*$ for nucleation of water on OTS coated surface in liquids (red markers), and $SR^*$ for nucleation at oil-air interface for some liquids (black markers).
Supplementary Figure S5: Cryo-FIB section of 1000 cSt Silicon Oil Surface
Supplementary Figure S6: Cryo-FIB section of 1000 cSt Silicon Oil Surface
Supplementary Figure S7: Polyhedral droplet profiles of condensing droplets. The image represents Frame 298/300 of the movie S3 accompanying this work. The droplets have been overlaid with polygons to represent the boundaries between condensing droplets.
Supplementary Note 1: Nucleation States and associated Free Energy of Nucleation

In state I, for a nuclei to form on a solid surface in presence of air environment, the total surface energy of the system is given by $E_{\text{drop \ a}} = \gamma_{ws} A_{ws}^d + \gamma_{wa} A_{wa}^d - \gamma_{sa} A_{sa}^d$. The superscript ‘$d$’ signifies droplet. The droplet volume is given by $V = \pi \psi_1 R^3/3$ where $R$=Radius of curvature of droplet, $\psi_1 = \left(2 + \cos \theta_{\text{ws}(d)}\right)\left(1 - \cos \theta_{\text{ws}(d)}\right)^2$ and $\theta_{\text{ws}(d)}$ is the contact angle of the condensate on the solid surface.

Since $\gamma_{ws} = \gamma_{sa} - \gamma_{wa} \cos \theta_{\text{ws}(d)}$, $A_{ws}^d = \pi R^2 \sin^2 \theta_{\text{ws}(d)}$ and $A_{wa}^d = 2 \pi R^2 \left(1 - \cos \theta_{\text{ws}(d)}\right)$ and $A_{sa}^d = A_{ws}^d$.

\[ E_{\text{drop \ a}} = \gamma_{sa} A_{ws}^d - \gamma_{wa} A_{wa}^d \cos \theta_{\text{ws}(d)} + \gamma_{wa} A_{wa}^d - \gamma_{sa} A_{ws}^d = \gamma_{wa} \left( A_{wa}^d - A_{ws}^d \cos \theta_{\text{ws}(d)} \right) \]

\[ \therefore E_I = E_{\text{drop \ a}} = \psi_1 \pi R^2 \gamma_{wa} = \psi_1 \pi \left( \frac{3V}{\pi \psi_1^2} \right)^{2/3} \gamma_{wa} \tag{1} \]

In state II, the surface energy required for creating a cluster through homogeneous nucleation within an oil environment is given as $E_{\text{II}} = \gamma_{ws} A_{ws}^o = 4 \pi \left(3V/4 \pi\right)^{2/3} \gamma_{wo}$.

In state III, we consider heterogeneous nucleation within oil. Oils for which the spreading coefficient of oil on surface in presence of water is positive i.e. $S_{\text{ot(w)}} = \gamma_{sw} - \gamma_{wo} - \gamma_{os} > 0$, heterogeneous nucleation of oil on solid surface is improbable because any molecule reaching the surface will be replaced by the surrounding oil molecules. Here, $\gamma_{sw}$ and $\gamma_{os}$ are the specific surface energy of solid in presence of condensate (water), and in presence of oil respectively. If the oil does not spread on the solid surface in presence of water (i.e. $S_{\text{ot(w)}} < 0$), the surface energy term is given by $E_{\text{drop \ o}} = \gamma_{ws} A_{ws}^o + \gamma_{wo} A_{wo}^o - \gamma_{so} A_{so}^o$.

Similar to derivation of State I, the surface energy can be written as
\[ E_{III} = E_{\text{drop,o}} = \psi_2 \pi \left( \frac{3V}{\pi \psi_2} \right)^{2/3} \gamma_{wo} \]

where \( \psi_2 = \left( 2 + \cos \theta_{wo(a)} \right) \left( 1 - \cos \theta_{wo(a)} \right)^2 \) and \( \theta_{wo(a)} \) is the contact angle of the condensate on the solid surface in presence of oil.

Finally we consider heterogeneous nucleation on oil-air interface (State \( IV \)). If the condensate itself does not wet the oil i.e. \( S_{wo(a)} = \gamma_{oa} - \gamma_{wa} - \gamma_{wo} < 0 \), and the oil does not cloak the condensate i.e. \( S_{ow(a)} < 0 \) then the nuclei will form a lens without any adsorbed monolayer on the condensate.\(^{10}\) Since the gravity effects are negligible for droplets that are much smaller than the capillary length (~ mm), the oil surface is effectively planar up to the three phase contact line for non-cloaking oil\(^\text{10}\) and the total surface energy of the system is given by \( E_{\text{lens}} = \gamma_{wo} A_{wo}^l + \gamma_{wa} A_{wa}^l - \gamma_{oa} A_{oa}^l \). The superscript ‘\( l \)’ signifies lens. By balance of forces at the three-phase contact line, we have \( \gamma_{wo} \sin \theta_{wo} = \gamma_{wa} \sin \theta_{wa}, \gamma_{wa} \cos \theta_{wa} + \gamma_{wo} \cos \theta_{wo} = \gamma_{oa}, \)
\( R_l = R_{wo} \sin \theta_{wo} = R_{wa} \sin \theta_{wa} \). Here: \( R_l \) is the base radius of the lens, \( R_{wo} \) is the radius of curvature of the lower section of lens, and \( R_{wa} \) is the radius of curvature of upper section of lens.

Also, \( \cos \theta_{wo} = \frac{\gamma_{oa}^2 + \gamma_{wo}^2 - \gamma_{wa}^2}{2 \gamma_{wa} \gamma_{wo}} \) and \( \cos \theta_{wa} = \frac{\gamma_{oa}^2 + \gamma_{wo}^2 - \gamma_{wa}^2}{2 \gamma_{wa} \gamma_{oa}} \)

Volume of lens, \( V = \frac{\pi}{3} \lambda R_l^3 \) where \( \lambda = \frac{\sin \theta_{wo} \left( 2 + \cos \theta_{wo} \right)}{(1 + \cos \theta_{wo})^2} + \frac{\sin \theta_{wa} \left( 2 + \cos \theta_{wa} \right)}{(1 + \cos \theta_{wa})^2} \) \( \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \ quadr
\[ A_{\text{wa}} = \frac{2\pi R_{\text{wa}}^2}{1 + \cos \theta_{\text{wa}}} = 2\pi R_{\text{wa}}^2 (1 - \cos \theta_{\text{wa}}) = 2\pi R_{\text{wa}}^2 (1 - \cos \theta_{\text{wa}}) \frac{\sin^2 \theta_{\text{wa}}}{\sin^2 \theta_{\text{wa}}} \]

\[
\therefore E_{\text{wa}} = \gamma_{\text{wa}} 2\pi R_{\text{wa}}^2 (1 - \cos \theta_{\text{wa}}) + \gamma_{\text{wa}} 2\pi R_{\text{wa}}^2 (1 - \cos \theta_{\text{wa}}) \frac{\sin^2 \theta_{\text{wa}}}{\sin^2 \theta_{\text{wa}}} - \gamma_{\text{wa}} \pi R_{\text{wa}}^2 \sin^2 \theta_{\text{wa}}
\]

\[ \Rightarrow E_{IV} = \pi \left(\frac{3V}{\pi \lambda}\right)^{2/3} \xi \gamma_{\text{wa}} \]

where \( \xi = \frac{2}{1 + \cos \theta_{\text{wa}}} - \cos \theta_{\text{wa}} \) and

\[ \lambda = \frac{\sin \theta_{\text{wa}} (2 + \cos \theta_{\text{wa}})}{(1 + \cos \theta_{\text{wa}})^3} + \frac{\sin \theta_{\text{wa}} (2 + \cos \theta_{\text{wa}})}{(1 + \cos \theta_{\text{wa}})^3} \cdot \]

Using \( \gamma_{\text{wa}} \sin \theta_{\text{wa}} = \gamma_{\text{wa}} \sin \theta_{\text{wa}} \), the surface energy can also be written in terms of oil-air surface tension as

\[ \Rightarrow E_{IV} = \pi \left(\frac{3V}{\pi \lambda}\right)^{2/3} \xi \frac{\sin \theta_{\text{wa}}}{\sin \theta_{\text{wa}}} \gamma_{\text{wa}} \]
Supplementary Note 2: Nucleation Rates in immiscible liquids

To understand the role of environment and miscibility of vapor molecules with the oil on nucleation rate, we consider the simpler case of homogeneous nucleation.

From nucleation theorem, the nucleation rate \( J \left( m^3 s^{-1} \right) \) for homogeneous nucleation is given by\(^{11} \)

\[
J = Z f^e C_0 \exp\left(-W^*/kT\right)
\]

(6)

where \( Z \) is the Zeldovich non-equilibrium factor, \( f^e \) is the collision rate of monomers with the critical cluster or the attachment frequency, \( C_0 \) is the concentration of nucleation sites that is equal to total number of molecules present in an environment, \( W^* \) is the work of formation \( (W^* = 4\beta^3/27\alpha^3) \) of critical cluster size \( n^* \left( \text{where } n^* = (2\beta/3\alpha)^3 \right) \). Here, \( \alpha = kT \ln(SR) \) and \( \beta = \pi \varphi \left( (3v_m)/(\pi \psi) \right)^{2/3} \). \(^{11}\) The Zeldovich factor \( Z \) is given by

\[
Z = \sqrt{\frac{-d^2W/dn^2}{2\pi kT}} = \frac{3\alpha^2}{4\beta^2} \sqrt{\frac{\beta}{\pi kT}}
\]

(7)

The attachment frequency \( f^e \) is dependent upon the saturation dynamics of condensing molecules with its environment that can be understood in terms of solute-solvent interactions. The transport of vapor molecules (solute) in a gaseous environment (solvent) occurs through diffusion and is given through Fick’s law. The transport of vapor molecules across the oil can also be expressed in terms of Fickean Diffusion if the passage of such molecules does not lead to anomalous effects such as swelling of the oil.\(^{12}\) With these assumptions, the attachment frequency \( f^e \) in a solvent (oil or gas) is given by\(^{11}\)
\[ f^* = \left(48 \pi^3 v_m \right)^{1/3} \alpha_m D_{ab} SR \cdot C_s \cdot n^{4/3} \] 

(8)

where \( D_{ab} \) is the mutual diffusion coefficient of the vapor in the solvent medium, and \( SR \) is the saturation ratio defined as \( SR = C_c / C_s \) where \( C_c \) is the solute concentration and \( C_s \) is the solubility of the solute molecule in the solvent, and \( \alpha_m \) is the monomer sticking coefficient (accommodation coefficient). When the solvent environment is a gas phase, then \( C_c = \frac{P_v N_A}{RT} \) and \( C_s = \frac{P_i N_A}{RT} \) from ideal gas law and the saturation ratio \( SR = C_c / C_s \) reduces to \( SR = P_v / P_i \). Here \( P_i \) is the saturation pressure at temperature \( T \), \( P_v \) is partial pressure of condensing species, \( N_A \) is the Avogadro’s number and \( \bar{R} \) is universal gas constant.

From Eqn. to , we can estimate nucleation rates of a condensate in different oils as a function of their properties such as solubility, diffusion coefficient of vapor molecules in oils etc. For comparison, we also evaluate nucleation rates of homogeneous nucleation in air. For water vapor diffusing in air the diffusion coefficient is \( \sim 10^{-5} \) m²/s and from ideal gas law, the typical molar concentration of water vapor molecules is \( \sim 1 \) moles/m³ at room temperatures. In ambient conditions, the molar concentration of gas molecules is \( \sim P_a / \bar{RT} \approx 40 \) moles/m³ where \( P_a \) is atmospheric pressure, from which the concentration of nucleation sites/volume is \( C_0 \approx 40N_A \).

The concentration of nucleation sites per unit volume in oils can be calculated from \( C_0 = (\rho_o / M_o + SR \cdot C_s) \cdot N_A \). Here \( \rho_o \) and \( M_o \) are the density and molecular weight of the oil respectively. As test oils for our...
theoretical analysis, we consider Silicone oil with viscosity 10 cSt \((\rho_o=935 \text{ kg/m}^3, M_o=1250)\) and Krytox \((\rho_o=1860 \text{ kg/m}^3, M_o=2400)\) since both polymers are considered very immiscible with water and also cloak the water drops. The diffusion coefficient of small molecules like water in polymers can be obtained using the Free-Volume theory, and is strongly a function of the solute concentration within the polymer.\(^{14,15}\) However, for simplification, we use diffusion coefficients of water vapor obtained by experimental data. The diffusion coefficient of water vapor in Silicone oil (PDMS)\(^{16}\) and Krytox like perfluoropolyether oil\(^8\) has been determined as \(\sim10^{-9} \text{ m}^2/\text{s}\) and \(\sim10^{-10} \text{ m}^2/\text{s}\) respectively. Although considered immiscible, the solubility of water in PDMS is \(\sim40 \text{ (mol of water)/(m}^3\text{ of polymer)}\), while the solubility of water in Krytox is \(\sim1.8 \text{ (mol of water)/(m}^3\text{ of polymer)}\).

Based on the above-mentioned data, we calculated the effect of saturation ratio on nucleation rate of water in environment of Silicone oil, Krytox and air (Fig. S4). In our calculations, we assumed that the accommodation coefficient is one in different environments. Surprisingly, we find that despite very low solubilities of water in Krytox and Silicone oil, these solubilities are still large enough so that large nucleation rates can be initiated at smaller saturation ratios within these oils as compared to supersaturation required to observe similar nucleation rates in air. This is attributed predominantly to the presence of large number of nucleation sites \((C_0)\) within the oils, and lowering of the interfacial tension of condensing vapor in these oils because of which lesser work is required to form the cluster. However as saturation ratio is increased, the nucleation rate in air can eventually surpass the nucleation rate in the oils.
Supplementary Figure S8: Nucleation rate versus saturation ratio for homogeneous nucleation within Silicone oil, Krytox and air.

The nucleation rate as shown in Fig. S8, was calculated under the assumption that the monomer sticking coefficient (accommodation coefficient) is one. Depending upon the accommodation coefficient, the supersaturation for obtaining large nucleation rates may vary considerably. Fig. S9(a) shows the variation of supersaturation with the accommodation coefficient to obtain large nucleation rate ($10^{10}$ drops/m$^3$/s). Studies on determining the sticking coefficient of vapor molecules on aerosols covered with films of organic liquids have shown that the sticking coefficient can be substantially lower than one for long chain organic liquids.$^{17,18}$ Although, the accommodation coefficient of vapor molecules in polymers like Silicone Oils or Krytox are not available, the behavior of water molecules in these oils can be
expected to be similar to long chain organic liquids. In such situation, it is quite possible that very high supersaturations may be required to form large number of nuclei in these oils.

It should also be noted that with respect to nucleation of water, miscibility matters greatly in initiating large nucleation rates. As an example, we show in Fig. S9(b) the Saturation Ratio required to cause homogeneous nucleation rate of $10^{30}$ drops/m$^3$/s. We observe that below solubility of 1 mol/m$^3$, the requirements for saturation ratio undergo sharp transitions and very large supersaturations are required.

**Supplementary Figure S9(a):** SR versus accommodation coefficient to obtain nucleation rate of $J=10^{30}$ drops/m$^3$/s for 2 nm diameter sized droplets. The orange line corresponds to oils with same properties as Krytox. The blue line corresponds to oils with same properties as Silicone Oil.

**Supplementary Figure S9(b):** SR versus solubility of water in different oils to obtain nucleation rate of $J=10^{30}$ drops/m$^3$/s for 2 nm diameter sized droplets. The violet line corresponds to oils with same molecular weight as Krytox. The blue line corresponds to oils with same molecular weight as Silicone Oil.
Supplementary Note 3: Permeation through thin films

The permeability ($P$) through an oil film is related with the diffusion coefficient ($D$) and solubility of solute species ($S$) as $P = DS$.$^{19-21}$ To examine the transport across submicroscopic thin oil films, we choose a system comprising of droplet on a LIS, taking advantage of the fact that such thin films (~$\mathrm{O(100 \text{ nm and less})}$) can form spontaneously on droplets if the impregnating oil spreads on the droplet (Fig. 4). Studying permeation through thin films during condensation is challenging since, nucleation at the oil-air interface is unavoidable. Instead of condensation, studying the evaporation of cloaked droplets in low humidity environment is advantageous since a concentration gradient automatically exists across the cloaked film, and the droplet acts as constant source of solute species to transfer across the cloak. As test liquids for cloaking the droplets, silicone oils of different viscosities (10, 100 and 1000 cSt viscosity) were chosen. For polymers with different viscosities, the diffusion coefficient of a solute has a negligible dependence on the polymer molecular weight (i.e. viscosity) at low mass fractions of the solute.$^{14, 22}$ However, the solubility of solute molecules is dependent upon the size of the solute and solvent molecule size$^{23}$, and hence expected to differ in different viscosity oils with identical chemical nature. A difference in evaporation behavior of a droplet on LIS with different viscosity polymer liquids is thus an indicator that the transport (permeation) of solute molecules is limited by the solute content within the oil film i.e. the solubility of the solute in the polymer oil.
To test this aspect qualitatively, LIS was prepared by impregnating a microtextured surface \((a=b=h=10\,\mu\text{m})\) with silicone oils of different viscosities (10, 100 and 1000 cSt viscosity). A 2 \(\mu\text{l}\) water droplet was gently deposited on the LIS samples and allowed to evaporate in room conditions. The room temperature and humidity were measured as 20 °C and 30% respectively. Evaporation of the droplet was observed using Zeiss AxioZoom microscope fitted with a ‘Plan APO-Z 1.5x lens’ and a polarizer at 260x magnification. The videos were recorded using Nikon D-800 camera (1920X1080) at 30 fps. The experiments showed that the droplet evaporation time increased on LIS as the viscosity of the impregnating liquid increased (Fig. S10, also see Supporting Movie S2). As discussed before, the delay in evaporation times on different viscosity silicone oils shows that the limiting mechanism during droplet evaporation is the solubility of water molecules in the oil.

**Figure S10 | Evaporation of droplet on LIS.** Evaporation of 2 \(\mu\text{l}\) droplet on LIS prepared by impregnating the micropost surface with Tetradecane \((S_{\text{sw(a)<0}},\text{Tetradecane})\), Silicon Oil \((S_{\text{sw(a)>0}},\text{Silicon Oil})\) with viscosities of 10 and 1000 cSt. The micropost surface (similar as used for Fig. 3) was impregnated with tetradecane,
and silicone oils with 10, 100 (not shown in image) and 1000 cSt viscosities. Scale bar: 250 µm. For complete video, see supplementary movie S2.

To understand the extent of the permeation barrier provided by the oil cloak around the droplets, we performed an experiment of droplet evaporation on a LIS impregnated with a non-cloaking liquid. For this experiment, we chose Tetradecane as the non-cloaking liquid because of its low vapor pressure and low solubility of water with this oil (see Supplementary Table 3). As before, a 2 µl droplet was deposited on the Tetradecane LIS, and the droplet was allowed to evaporate under the same thermodynamic conditions, as were used for the droplet evaporation on silicone oil LIS experiments. Comparing the time scales of evaporation, we found that indeed the droplet evaporation time was shortened on the non-cloaking case. However, the evaporation times of the droplets on cloaking LIS of different viscosities were within similar order as magnitude as evaporation times on the non-cloaking LIS. This indicates that silicone oils used in this work may not act as significant barriers against permeation. Further, the evaporation time scales between different viscosity oils, though different, are within the same order of magnitude of each other. Based on this, one may surmise that the permeation rates across these oils are similar.
Supplementary Note 4: Droplet growth modeling

In general, the volume of droplet and the principal radius of growth are related as \( V = \pi \Psi R^3 / 3 \) from where \( R = \sqrt[3]{(3V) / (\pi \Psi)} \), \( \Psi \) is a shape factor depending upon the geometry of condensate.

For a droplet condensing in form of a lens on a oil or soft surface, the droplet growth occurs at the oil-air interface and the lens-air interface is the only interfacial area that is considered in contributing to the growth by direct diffusion from the air. The volume of the upper lens with the principle radius of curvature \( R=R_{\text{wa}} \), is

\[
V_{\text{wa}}' = \pi \Psi_{\text{wa}} R_{\text{wa}}^3 / 3, \text{ where } \Psi_{\text{wa}} = (2 + \cos \theta_{\text{wa}})(1 - \cos \theta_{\text{wa}})^2
\]

(9)

Following the method of Sokuler\textsuperscript{24}, the basic equation of a drop condensing on a surface is given as

\[
\frac{dV_{\text{wa}}'}{(V_{\text{wa}}')^{\frac{2}{3}}} = \left( \frac{3}{\pi \Psi_{\text{wa}}} \right)^\frac{1}{3} \frac{4 \pi D_{ab}}{\rho_w} (c_v - c_i) dt
\]

(10)

Here, \( c_v \) is the concentration of vapor far away from the droplet, and \( c_i \) is the concentration of vapor at the droplet interface, \( \rho_w \) is the density of the condensate and \( D_{ab} \) is the diffusion coefficient of the vapor in the medium (air). Eqn. can be rearranged to give

\[
\Rightarrow 3\frac{1}{2} \pi \Psi R^2 \frac{dR}{R_{\text{wa}} \left( \pi \Psi_{\text{wa}} \right)^\frac{1}{3}} = \left( \frac{3}{\pi \Psi_{\text{wa}}} \right) \frac{4 \pi D_{ab}}{\rho_w} (c_v - c_i) dt \quad \Rightarrow \Psi_{\text{wa}} R_{\text{wa}} dR_{\text{wa}} = \frac{4 D_{ab}}{\rho_w} (c_v - c_i) dt
\]

(11)

Assuming vapor to behave as ideal gas, the concentration \( c \) can be given as \( c = PM_w / \bar{R}T \). Here \( P \) is the pressure, \( M_w \) is the molecular weight of the condensate, \( \bar{R} \) is the gas constant and \( T \) is the temperature. Far away from the droplet, the vapor pressure is \( P = P_v \) and the vapor temperature \( T = T_v \). If the humidity far
away from the droplet (Eg. humidity of room if condensation is done in open air) is $RH$, then the vapor pressure $P_v$ can be given as $P_v = RH \cdot P_{saturation}(T_v)$ where $P_{saturation}(T_v)$ is the saturation vapor pressure at temperature $T_v$. At the droplet interface, the droplet is maintained at a subcooled temperature $T = T_i$.

Because of the droplet curvature, the droplet saturation vapor pressure can be given through Kelvin Equation and expressed as $P_i = P_{i0} \cdot \exp\left(\frac{2\gamma_{wa}V_m}{R_{wa}RT_i}\right)$ where $P_{i0} = P_{saturation}(T_i)$ is the saturation vapor pressure at temperature $T_i$. For a given subcooling, supersaturation conditions are generated around the interface, and depending upon the driving force ($SR = P_v/P_{i0})$, a minimum droplet size nucleates on the surface. The minimum droplet size is given by $R_0 = \frac{2\gamma_{wa}V_m \cos \theta}{RT_i \ln SR}$ so that

$$P_i = P_{i0} \cdot \exp\left[\frac{R_0 \ln(SR)}{R_{wa}}\right] = P_{i0} \cdot SR^{R_0/R_{wa}}.$$ With abovementioned relations, Eqn. can be given as

$$\Rightarrow \psi_{wa} R_{wa} dR_{wa} = \frac{4M_w D_{ab}}{\rho_w \psi_{wa} R} \left(\frac{P_i}{T_v} - \frac{P_{i0}}{T_i}\right) dt$$

(12)

Eqn. is the fundamental equation that can be solved for droplet growth for all size ranges. To get a simplified solution, we can solve Eqn. in limit of $R_{wa} >> R_0$. Under such assumption $P_i = P_{i0} \cdot SR^{R_0/R_{wa}} = P_{i0}$

$$\Rightarrow R_{wa} dR_{wa} = \frac{4M_w D_{ab}}{\rho_w \psi_{wa} R} \left(\frac{SR \cdot P_{i0}}{T_v} - \frac{P_{i0} \cdot SR^{R_0}}{T_i}\right) dt \Rightarrow R_{wa} dR_{wa} = \frac{4M_w D_{ab} \cdot SR \cdot P_{i0}}{\rho_w \psi_{wa} RT_i} \left(\frac{T_i}{T_v} - \frac{1}{SR}\right) dt$$

(13)

$$\Rightarrow R_{wa} dR_{wa} = \eta dt \quad \text{where} \quad \eta = \frac{4M_w D_{ab} \cdot P_{i0} \cdot SR \cdot \psi_{wa} \cdot RT_i}{\rho_w \psi_{wa} RT_i} \left(\frac{T_i}{T_v} - \frac{1}{SR}\right)$$

Eqn. can be integrated and simplified under the assumption of $R >> R_0$ to give evolution of the curvature of the upper segment of the lens as
\[ R_{wa} = \sqrt{R_o^2 + 2\eta t} \]  

(14)

Here, we note that a part of the volume addition due to condensation also increases the volume of the lower segment of the lens. As a result, the effective change in upper lens volume is decreased. If we assume that in a time \( dt \), the droplet grows from \( R_{wa} \) to \( R_{wa} + dR_{wa} \), then the volume added to the upper lens is \( dV_{wa}^l = 3\pi \psi_{wa} R_{wa}^2 dR_{wa} \). Assuming that this volume addition adjusts instantaneously throughout the droplet, the new effective curvature of the upper lens becomes \( R_{wa} + dR_{wa, eff} \) and the curvature of the lower lens becomes \( R_{wo} + dR_{wo, eff} \) that are related by \((R_{wo} + dR_{wo, eff}) \sin \theta_{wo} = (R_{wa} + dR_{wa, eff}) \sin \theta_{wa}\) from where we get \( dR_{wo, eff} \sin \theta_{wo} = dR_{wa, eff} \sin \theta_{wa} \). The newly adjusted volumes of the upper and lower part of lens are given \( dV_{wa, eff}^l = 3\pi \psi_{wa} R_{wa, eff}^2 dR_{wa, eff} \) and \( dV_{wo, eff}^l = 3\pi \psi_{wo} R_{wo, eff}^2 dR_{wo, eff} \) respectively. From mass conservation, we have then \( dV_{wa}^l = dV_{wa, eff}^l + dV_{wo, eff}^l \). From this we get,

\[
3\pi \psi_{wa} R_{wa}^2 dR_{wa} = 3\pi \psi_{wa} R_{wa, eff}^2 dR_{wa, eff} + 3\pi \psi_{wo} R_{wo, eff}^2 dR_{wo, eff}
\]

\[ \Rightarrow R_{wa}^2 dR_{wa} = \left[ 1 + \frac{\psi_{wa}}{\psi_{wo}} \left( \frac{\sin \theta_{wa}}{\sin \theta_{wo}} \right)^3 \right] R_{wa, eff}^2 dR_{wa, eff} \]

\[ \Rightarrow R_{wa, eff}^2 dR_{wa, eff} = \varphi R_{wa}^2 dR_{wa} \text{ where } \varphi^{-1} = 1 + \frac{\psi_{wa}}{\psi_{wo}} \left( \frac{\sin \theta_{wa}}{\sin \theta_{wo}} \right)^3. \]  

(15)

Here, the correction factor \( \varphi \), gives the effective decrease in radius of curvature. For \( \varphi = 1 \), the case of condensation on hard surfaces is recovered. Integrating Eqn. between limits of \( R_0 \) and \( R_{wa} \), we get

\[ \Rightarrow R_{wa, eff}^3 = (1 - \varphi) R_0^3 + \varphi R_{wa}^3 \]  

(16)

Combining Eqn. and , the effective growth rate of the droplet is found as
\[ R_{\text{wa, eff}}^3 = (1 - \phi) R_0^3 + \phi (R_0^2 + 2\eta t)^{3/2} \]  

(17)

For droplets of size $R >> R_0$, the contribution of the initial droplet size $R_0$ is negligible so that the droplet growth equation in terms of the upper lens curvature becomes

\[ R_{\text{wa, eff}} = \phi^{1/3} \sqrt[3]{2\eta t} \]  

(18)

For convenience, we use the symbol $R$ instead of $R_{\text{wa, eff}}$ to denote the droplet growth.

From here, the droplet growth rate ($U_d = dR/dt$) is given by $U_d = \phi^{1/3} \sqrt[3]{0.5\eta t^{-1/2}} = \phi^{2/3} \eta / R$. Note, here the singularity in droplet growth velocity at $t=0$ can be avoided by preserving the initial term $(R_0)$.
Supplementary Note 5: Drainage time estimation

For two droplets of the same size approaching each other with an applied force $F_{\text{ext}}$, the drainage of the intermediate fluid between them can be given through Stefan Reynolds flat plate model. The disjoining pressure $\Pi(t)$ has the non-retarded Van der Waals form: $\Pi(t) = -A/6\pi l^3$ where $A$ is the Hamaker constant.\(^{25}\) The drainage equation assuming that droplet surfaces are immobile is then given by\(^{25}\)

$$F_{\text{ext}} + \frac{3\pi \eta \alpha^4}{2l^3} \frac{dl}{dt} + \frac{\pi \alpha^2 A}{6\pi l^3} = 0$$

(19)

where $\alpha$ is the drop deformation and $l$ is the separation between the drops.

If two droplets are located initially at distance much larger than the range at which intermolecular forces between droplets become active, then it is important to estimate the $F_{\text{ext}}$ that pushes the droplet in proximity. If we consider the dynamics of growth of an isolated droplet in an outer fluid then a droplet growing at a rate of $U_d = dR/dt$ exerts a driving force on the surrounding oil that can be given by

$$F_{\text{ext}} = \mu_o R U_d$$

where $R = \phi^{1/3} \sqrt{2\eta}$ and $U_d = \phi^{2/3} \eta / R$. This implies that two droplets approach each other with a constant force $F_{\text{ext}} = \mu_e \eta \phi$ irrespective of the droplet size. In addition to the force exerted by growing droplets to the surrounding medium, neighboring droplets at a oil-air interface or sitting atop a LIS can also exert additional attractive force towards each other\(^{26}\) due to the interaction of their \textit{wetting ridges} that will affect the drainage time. Solving the dynamics of film drainage for condensing droplets at large separations requires further consideration and is out of the scope of this work.
If the droplets are separated by a distance ($l\sim100$ nm) where the drainage happens purely due to VdW forces, then in absence of an external force, Eqn. reduces to $\frac{dl}{dt} = -A / (9\pi\mu_a a^2)$. For nearly spherical droplets, the film radius or deformation $a$ can be given by $a \sim \sqrt{Rl}$.\textsuperscript{22} Substituting this we get $\frac{dl}{dt} = -A / (9\pi\mu_a R)$ and by integrating this, the coalescence time can be estimated as $t_{c,\text{New}} = 9\pi\mu_a RL^2 / 2A$. 
Reference: